MATH 2460 EXAM 2

NAME Key — GRADE ——— OUT OF 15 PTS

Answer the following questions (**2pts each**) correctly (**NO** decimal answer!) for a full credit.

(1point) 1. Evaluate $\int_0^1 xe^x\,dx$ (<u>**show**</u> work for a full credit!)

$u = x \quad du = 1$

$dv = e^x \quad v = e^x \quad \longmapsto xe^x - \int e^x dx = xe^x - e^x + c$

$= e^x(x-1) + c$

Now $\left[e^x(x-1)\right]_0^1 = 0 - (-e^0)$

$= e^0 = \boxed{1}$

2. Find the length of the curve $y = \frac{2}{3}(x^2+1)^{3/2}$ over $[0,3]$.

$L = \int_0^3 \sqrt{1+(y')^2}\,dx = \int_0^3 \sqrt{(2x^2+1)^2}\,dx$ since

$y' = 2x(x^2+1)^{1/2}$ and $1+(y')^2 = 1+4x^2(x^2+1) = 1+4x^2+4x^4 = (2x^2+1)^2$

So $L = \int_0^3 (2x^2+1)\,dx = \left[\frac{2x^3}{3} + x\right]_0^3 = \boxed{21}$

3. Set up and evaluate the definite integral for the area of the surface generated by revolving the curve $y = \frac{1}{3}x^3$ about the x-axis, with $0 \le x \le 3$. **Show** all your work to get a full credit!

$$S = 2\pi \int_0^3 f(x) \sqrt{1 + (f')^2}\, dx$$

$$y = \frac{1}{3}x^3 \rightarrow y' = x^2 \rightarrow (y')^2 = x^4 \rightarrow 1 + y'^2 = 1 + x^4$$

$$S = 2\pi \int_0^3 \frac{x^3}{3} \sqrt{1 + x^4}\, dx = \frac{2\pi}{3} \int_0^3 (1+x^4)^{1/2} \frac{1}{4} x^3\, dx$$

$$= \frac{\pi}{9} \left[(1+x^4)^{3/2} \right]_0^3$$

(or let $u = 1 + x^4$
$du = 4x^3 dx$
$\frac{1}{4} du = x^3 dx$)

$$= \frac{\pi}{9} \left[\sqrt{82^3} - 1 \right] \text{ unit}^2 \text{ or}$$

$$\frac{\pi}{9} \left[82\sqrt{82} - 1 \right]$$

4. Evaluate $\int_{-2\pi}^{2\pi} (sin^2 x + 1)\, dx$.(**show** work for a full credit!)

$$= \int_{-2\pi}^{2\pi} \left[\frac{1 - \cos 2x}{2} + 1 \right] dx.$$

$$= \int_{-2\pi}^{2\pi} \left[\frac{3}{2} - \frac{1}{2}\cos 2x \right] dx$$

$$= \left[\frac{3}{2} x - \frac{1}{4} \sin 2x \right]_{-2\pi}^{2\pi}$$

$$\boxed{= 6\pi}$$

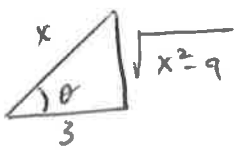
5. Evaluate $\int \frac{dx}{x^2\sqrt{x^2-9}}$

$x = 3\sec\theta \Rightarrow x^2 = 9\sec^2\theta$ also $dx = 3\sec\theta\tan\theta\, d\theta$

So we have $\int \frac{3\sec\theta\tan\theta\, d\theta}{9\sec^2\theta\sqrt{9(\sec^2\theta-1)}} = \frac{1}{3}\int \frac{\sec\theta\tan\theta}{\sec^2\theta\tan\theta}\, d\theta = \frac{1}{9}\int \frac{1}{\sec\theta}\, d\theta$

$= \frac{1}{9}\int \cos\theta\, d\theta = \frac{1}{9}\sin\theta + C$

Now



so $\sin\theta = \frac{\sqrt{x^2-9}}{x}$, Hence we have

$\sec\theta = \frac{x}{3} = \frac{hyp}{adj}$

$\frac{1}{9}\left(\frac{\sqrt{x^2-9}}{x}\right) + C$

6. Evaluate $\int \sqrt{4-x^2}\, dx$

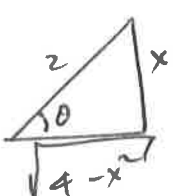
$x = 2\sin\theta \Rightarrow dx = 2\cos\theta\, d\theta$ and $x^2 = 4\sin^2\theta$

So $\int\sqrt{4-x^2}\, dx = \int\sqrt{4(1-\sin^2\theta)}\,(2\cos\theta\, d\theta)$

$= 4\int \cos\theta(\cos\theta)\, d\theta = 4\int \cos^2\theta\, d\theta$

$= 4\int\left(\frac{1+\cos 2\theta}{2}\right) d\theta$

$= 2\int (1 + \cos 2\theta)\, d\theta = 2\left(\theta + \frac{1}{2}\sin 2\theta\right) + C$



$= 2\theta + \sin 2\theta + C$

$\cos\theta = \frac{\sqrt{4-x^2}}{2}$

$\sin\theta = \frac{x}{2}$

$= 2\sin^{-1}\left(\frac{x}{2}\right) + 2\left(\frac{x}{2}\right)\left(\frac{\sqrt{4-x^2}}{2}\right) + C$

$\theta = \arcsin\left(\frac{x}{2}\right)$

$\boxed{\text{or } 2\arcsin\left(\frac{x}{2}\right) + \frac{1}{2}x\sqrt{4-x^2} + C}$

Note: $\sin 2\theta = 2\sin\theta\cos\theta$

7. Evaluate $\int \frac{1}{x^2 - 5x + 6}\, dx$

$$= \int \frac{1}{(x-3)(x-2)}\, dx = \int \left(\frac{1}{x-3} - \frac{1}{x-2} \right) dx$$

$$= \int \frac{1}{x-3}\, dx - \int \frac{1}{x-2}\, dx$$

$$= \ln|x-3| - \ln|x-2| + C \quad \text{or}$$

$$\ln\left|\frac{x-3}{x-2}\right| + C$$

Not ASKED

8. Evaluate $\int \frac{1}{x^2 - 2x + 5}\, dx$

ASKED!

#8 $\int \frac{2x+1}{x^2-7x+12}\, dx$ (asked!) (See Note.)

$$\frac{2x+1}{x^2-7x+12} = \frac{A}{x-3} + \frac{B}{x-4}$$

$$\Rightarrow \frac{2x+1}{x^2-7x+12} = \frac{2x+1}{(x-3)(x-4)} = \frac{A(x-4)}{(x-3)(x-4)} + \frac{B(x-3)}{(x-3)(x-4)}$$

$$\Rightarrow 2x+1 = A(x-4) + B(x-3)$$

when $x = 3 \longrightarrow A = -7$ and when $x = 4 \longrightarrow B = 9$

So $\int \frac{2x+1}{x^2-7x+12}\, dx = \int \frac{-7}{x-3}\, dx + \int \frac{9}{x-4}\, dx$

$$\boxed{\int \frac{2x+1}{x^2-7x+2}\, dx = -7\ln|x-3| + 9\ln|x-4| + C}$$